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On the Mode of Transmission of Heat Through Air.

A. Fundamental Theory.

1. Introduction:-

The problem to be considered in this paper is the general mode of heat transfer through air which is entirely enclosed within rigid boundaries, where the boundaries are maintained at some fixed temperature distribution. The following analysis may equally well apply to any fluid, particularly a gas. For this reason, the medium shall simply be referred to as a "fluid". Furthermore, it is customary to speak of a "heat" transfer, which it ultimately is; for this reason this term has been used in the title. Nevertheless, in studying the details of this problem it will be more exact to speak of an "energy"transfer. The first law of thermodynamics deals with the conservation of "energy", and this law will be applied in the analysis. The term "energy" will be used, unless for any reason the name "heat" may be found more convenient, and then each case will be discussed separately.

From a physical conception of a system, such as is to be considered, it may be said that there will be a resultant energy transfer from those portions of the boundary which are at a higher temperature to those at a lower temperature, and the three modes of transfer will be by radiation, conduction, and convection.

The laws of radiation are well established, and if the character and geometry of the bounding surfaces are known, and also their abso-

lute temperatures, the amount of energy transmitted by radiation can be determined. For gases, the absorption of this energy is slight, and the gaseous medium will play but a minor part in this mode of energy transfer. For liquids the absorption may be appreciable. However, this problem is to be limited to ordinary temperatures, and to a study of conduction and convection only. If radiation is appreciable, it is to be subtracted from the total energy transmitted.

By "conduction" in any given direction at any point in the fluid shall be meant the resultant energy that is transmitted in this direction by the individual molecules in the neighborhood of this point due to their random motion. This will be analogous to the mode of transmission through a solid. Following the analogy of a solid, consider energy conducted across any surface at which a temperature-gradient exists, and the rate of energy transfer to be proportional to the negative of this gradient. If h_1 is the energy conducted across unit area in unit time, then, by definition,

$$h_1 = -\mathbf{K} \nabla \theta \cos (\bar{n}_1 \nabla t) \qquad (1)$$

where ∇t is the temperature-gradient, and $\bar{n}$ a unit vector normal to the surface. $\mathbf{K}$ is a proportionality factor, and may be a function of the temperature. $\mathbf{K}$ will be referred to as the "conductivity". In this connection it may be said that if the temperature at each point within the fluid is constant, a conduction of "energy" can also be properly spoken of as a conduction of "heat". However, even for the case of a solid, if there are fluctuations of temperature at each point

point there will be expansions or contractions within the material, which will constitute a transfer of mechanical energy. To include both forms one must employ the term "energy".

By "convection" in any given direction at any point in the fluid shall be meant the energy that is conveyed in this direction by the gross currents within the neighborhood of this point. This form of transfer is composed of two separate effects: (a) the transfer of the energy contained within the molecules, and (b) the transfer of mechanical energy that arises from changes in the specific volume of the fluid. If ϵ is the internal energy of unit mass of fluid, p the pressure at a given point, and Ω the specific volume of the fluid at this point, then by the first law of thermodynamics

$$d\epsilon = \Delta q - p\,d\Omega \qquad (2)$$

where Δq is the "heat" added to this unit mass.

The following relations will always hold:

$$\Delta q = C_p d\theta - \theta \left(\frac{\partial\Omega}{\partial\theta}\right)_p dp \qquad (3)$$

and

$$d\Omega = \left(\frac{\partial\Omega}{\partial\theta}\right)_p d\theta + \left(\frac{\partial\Omega}{\partial p}\right)_\theta dp \quad . \qquad (4)$$

Therefore,

$$\epsilon = \int_{\theta_0}^{\theta}\left[C_p - p\left(\frac{\partial\Omega}{\partial\theta}\right)_p\right] d\theta - \int_{p_0}^{p}\left[\theta\left(\frac{\partial\Omega}{\partial\theta}\right)_p + p\left(\frac{\partial\Omega}{\partial p}\right)_\theta\right] dp \qquad (5)$$

where (p_0, θ_0) is an arbitrary condition for the fluid, and C_p the specific heat at constant pressure. If ρ is the density of the fluid at this point, and V the vector velocity, the "convection" h_c per unit area per unit time across any element of surface at this point will be defined by the equation

$$h_c = \rho V \epsilon \cos(\bar{n}, V) \qquad (6)$$

where $\dot{\epsilon}$ is given by equation (8).

A term which will be used quite frequently is "steady state". The system will be considered in a steady state if the temperature, pressure, and vector velocity at each point within the space and at the boundaries are constant for all times.

Thus the problem to be considered may be stated as follows: Given a fluid of certain physical properties enclosed by rigid boundaries, the latter also having definite physical properties (as conductors or insulators), with a fixed temperature distribution over these boundaries. To find, in a steady state, the values of h_1 and h_2 over any given surface within the enclosed medium, or at any portion or portions of the boundary.

2. General Equations of Motion:

Consider the forces that cause circulation, or the gross velocities. There will, of course, be gravitational forces which may be written as R (X,Y,Z) per unit mass. If there is any variation of pressure within the space, there will be the force $-\nabla p$ per unit area. In a steady state these forces will be balanced by the opposing forces of viscosity. Let

V (u,v,w) = vector velocity of the fluid at a given point

p = pressure at that point
θ = absolute temperature at that point
ρ = density " " " "
η = coefficient of viscosity.

The equations of motion* for this case are:

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* Lamb's Hydrodynamics, 1916 edition; p. 573

$$\left.\begin{aligned}
\rho \frac{Du}{Dt} &= \rho X - \frac{\partial p}{\partial x} + \frac{1}{3}\eta \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + \eta \nabla^2 u \\
\rho \frac{Dv}{Dt} &= \rho Y - \frac{\partial p}{\partial y} + \frac{1}{3}\eta \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + \eta \nabla^2 v \\
\rho \frac{Dw}{Dt} &= \rho Z - \frac{\partial p}{\partial z} + \frac{1}{3}\eta \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + \eta \nabla^2 w
\end{aligned}\right\} \quad (7)$$

where $\nabla^2(\ ) = \frac{\partial^2(\ )}{\partial x^2} + \frac{\partial^2(\ )}{\partial y^2} + \frac{\partial^2(\ )}{\partial z^2}$

In the ~~Einstein~~ Eulerian system of transformation, the total derivative may be written as

$$\frac{D(\ )}{Dt} = \frac{\partial(\ )}{\partial t} + u\frac{\partial(\ )}{\partial x} + v\frac{\partial(\ )}{\partial y} + w\frac{\partial(\ )}{\partial z}, \qquad (8)$$

and in a steady state $\frac{\partial(\ )}{\partial t} = 0$. Introducing this into (7), the equations of motion become

$$\left.\begin{aligned}
\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) &= \rho X - \frac{\partial p}{\partial x} + \frac{1}{3}\eta\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + \eta\nabla^2 u \\
\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) &= \rho Y - \frac{\partial p}{\partial y} + \frac{1}{3}\eta\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + \eta\nabla^2 v \\
\rho\left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) &= \rho Z - \frac{\partial p}{\partial z} + \frac{1}{3}\eta\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + \eta\nabla^2 w
\end{aligned}\right\} \quad (9)$$

3. Equations for the Continuity of Matter and Energy.

For the continuity of matter may be employed the well known equation

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \qquad (10)$$

Consider the continuity of energy. In a steady state the temperature at each point will be some fixed value. Also, there will be no

source or sink within the fluid; i.e. the rate of flow of energy into any element of volume will equal the rate flowing out of this element. Consider any finite volume within the medium. The total gain of energy by this volume during a steady state will be

$$\int (h_1 + h_2)\, d\tau = \int \left\{ - K \nabla_n \theta + \rho V_n \, \epsilon \right\} dS$$

$$= \int \operatorname{div} \left\{ - K \nabla \theta + \rho V \epsilon \right\} d\tau = 0 \qquad (11)$$

where ϵ is given by equation (5). Since this equation is true for any finite volume, the integral will be zero, and

$$\operatorname{div} \left\{ - K \nabla \theta + \rho V \epsilon \right\} = 0 \qquad (12)$$

This equation will be referred to as the equation for the continuity of energy.

4. Resumé of Equations.

The solution of the given problem reduces to a solution of the following set of fundamental equations.

Equations of motion:-

$$\left.\begin{aligned}
\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) &= \rho X - \frac{\partial p}{\partial x} + \frac{1}{3} \eta \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) + \eta \nabla^2 u \\
\rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) &= \rho Y - \frac{\partial p}{\partial y} + \frac{1}{3} \eta \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) + \eta \nabla^2 v \\
\rho \left(u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) &= \rho Z - \frac{\partial p}{\partial z} + \frac{1}{3} \eta \frac{\partial}{\partial z} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) + \eta \nabla^2 w
\end{aligned}\right\} \quad \text{I}$$

Continuity of Matter:-

$$\operatorname{div} (\rho V) = 0 \qquad (\text{II})$$

Continuity of Energy:

$$\operatorname{div} \left\{ - K \nabla \theta + \rho V \epsilon \right\} = 0 \qquad (\text{III})$$

Equation of State:-

$$p = F (\theta , \Omega) \qquad (\text{IV})$$

If there is any flow at a rigid boundary, it will be tangential to the surface, or

$$\ell u + mv + nw = 0 \qquad (V)$$

where (ℓ, m, n) are the direction cosines of the normal to the surface.

The general solution of these equations will consist in obtaining a relation between p, ρ, θ, u, v, w in terms of x,y,z. Knowing this, one can determine h_1 and h_2 at each point within the space. Further, if a surface is given, the integral $\int h_1 \, dS$ and $\int h_2 dS$ can be determined, which gives the quantity of energy conducted and convected across this surface under the given conditions.

5. Relation between Velocity and Temperature-Gradient:-

An interesting relation can be obtained from equations (II) and (III) of section 4. Rewriting these equations,

$$\operatorname{div} (\rho V) = 0 \qquad (13)$$

$$\operatorname{div} \left\{ - K\nabla \theta + \rho V \int_{\theta_0}^{\theta} \left[C_p - p \left(\frac{\partial \Omega}{\partial \theta} \right)_p \right] d\theta - \rho V \int_{p_0}^{p} \left[\theta \left(\frac{\partial \Omega}{\partial \theta} \right)_p + p \left(\frac{\partial \Omega}{\partial \theta} \right)_\theta \right] dp \right.$$

Expanding (13), and substituting it into equation (14), one obtains

$$\operatorname{div} (- K\nabla \theta) + \rho \left(V. \operatorname{grad} \int_{\theta_0}^{\theta} \left[C_p - p \left(\frac{\partial \Omega}{\partial \theta} \right)_p \right] d\theta \right)$$

$$- \rho \left(V. \operatorname{grad} \int_{p_0}^{p} \left[\theta \left(\frac{\partial \Omega}{\partial \theta} \right)_p + p \left(\frac{\partial \Omega}{\partial p} \right)_\theta \right] dp \right) = 0 \qquad (15)$$

where the second and third terms are the density times the scalar product of V and the gradient of the integral.

The writer has in mind the application of this theory to the case of air. For air, or any of the permanent gases, the perfect gas equation will hold very approximately:

viz. $\quad p \Omega = R \theta \qquad (16)$

This may be simplified still further for the case under consideration. The pressure variations within the space will be due to different heights of air columns, and if spaces are considered of such size as can be handled in a laboratory, these variations will be small compared to the absolute pressure. Thus, a fair approximation can be made by assuming the pressure throughout the space as being constant. If the air within the enclosure is connected by some small channel to the atmosphere, the inside pressure will, during a steady state, be equal to atmospheric pressure.

Thus, from (16)

$$\rho = \left(\frac{p}{R} \right) \frac{1}{\theta} = \frac{M}{\theta} \qquad (17)$$

Introducing (17) into equation (15), and considering the pressure is constant, the equation reduces to the form

$$(V \cdot \nabla \theta) = \frac{\theta \operatorname{div} (\kappa \nabla \theta)}{M (C_p - R)} \qquad (18)$$

The component of the velocity parallel to the temperature-gradient is

$$|V| \cos (V, \nabla \theta) = \frac{\theta \operatorname{div} (\kappa \nabla \theta)}{|\nabla \theta| M (C_p - R)} \qquad (19)$$

With the aid of equation (19) one can determine the velocity parallel to the temperature-gradient. However, it does not yield a value for the vector velocity, unless the value of $\cos (V, \nabla \theta)$ is given.

6. Fundamental Equations in Two Dimensions.

Using the approximation of equation (17), and considering only the case of two dimensions, the fundamental equations reduce to

a somewhat simpler form, as follows:-

Equations of motion:-

$$\left.\begin{aligned}\frac{K}{\theta}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right) &= \frac{K}{\theta}X+\frac{1}{3}\eta\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\eta\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)\\ \frac{K}{\theta}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right) &= \frac{K}{\theta}Y+\frac{1}{3}\eta\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\eta\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)\end{aligned}\right\}\quad(20)$$

Continuity of Matter:-

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=\frac{1}{\theta}\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right)\qquad(21)$$

Continuity of Energy:-

$$\frac{\partial}{\partial x}\left(\kappa\frac{\partial\theta}{\partial x}\right)+\frac{\partial}{\partial y}\left(\kappa\frac{\partial\theta}{\partial y}\right)=K(C_p-R)\left\{\frac{\partial}{\partial x}\left[u\left(1-\frac{\theta_0}{\theta}\right)\right]+\frac{\partial}{\partial y}\left[v\left(1-\frac{\theta_0}{\theta}\right)\right]\right\}\quad(2$$

Substituting (21) into (22), one obtains

$$\frac{\partial}{\partial x}\left(\kappa\frac{\partial\theta}{\partial x}\right)+\frac{\partial}{\partial y}\left(\kappa\frac{\partial\theta}{\partial y}\right)=\frac{K(C_p-R)}{\theta}\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right)\qquad(23)$$

Equations (20) and (23) may be considered the fundamental equations in two dimensions.

7. Effect of a Rigid Boundary.

At a perfectly rigid and smooth boundary the normal velocity will be zero. If the boundary is a perfect insulator, there will be no heat-flow normal to the surface. Therefore, for a perfect insulator, isothermal lines meet the surface at normal incidence. For the case of a perfect conductor, the temperature gradient will be normal to the surface, and therefore the surface of a perfect conductor will be one of the families of isothermal surfaces. These two physical laws can guide one in constructing isothermal surfaces.

I. Experimental Results.

It would be highly desirable to obtain a solution of this problem on a mathematical basis. But this is a formidable task, and can hardly be hoped for at the present time. Experiment seems the only means available, and it was with this in mind that the writer has undertaken this work.

1. Description of Apparatus:

A very simple system is chosen, which consists of two square parallel plates with their corresponding edges parallel. They are arranged so that the distance between the plates is maintained constant, while they can be rotated as a unit about a horizontal axis parallel to one edge. With this arrangement symmetry is obtained about a vertical plane normal to the axis of rotation and passing through the center of the plates. Thus, the system lends itself to a study of the problem in two dimensions.

In general design and construction the apparatus is virtually that described by Mr. M.S.VanDusen.* For details of construction the reader may be referred to this paper. Figure I is a sketch of the apparatus as employed in these experiments. A is a flat electric heater made by winding constantan ribbon diagonally on a sheet of fibre board 24 inches square. The winding is arranged so that the lengths of ribbon on each face of the fibre are equally spaced and parallel to

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* Journ. Amer. Soc. Heat. & Ventil. Eng., vol. 26, 7; Oct., 1920

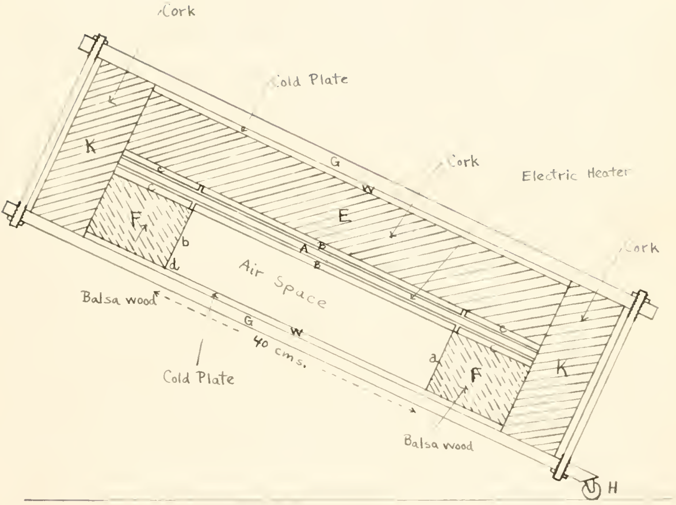


Figure 1.

Sketch of Conductivity Apparatus.

each other, and perpendicular to the corresponding lengths on the other side of the fibre board. The resistance per unit area of heater is thus the same over the whole area, except around the edges which are for away from the region in which uniform heating is essential, and will therefore not affect the results.

On each side of this heater are placed a center copper plate B,B, 40 cms. square, with a border C,C, also of copper, whose outside dimensions are 24 inches square and which is separated from the center plate by an air space $\frac{1}{8}$ inch wide. These plates are $\frac{5}{32}$ inches thick. One of the center plates is nickel plated and highly polished on that side opposite from the electric heater. The other surfaces have the ordinary finish as given to rolled sheet copper. These copper plates are insulated from the heating coil by **thin** sheets of mica. A small additional heater is placed around the perimeter of the main heater [illegible], which consists simply of asbestos-covered constantan wire wound several times around the perimeter of the fibre board. The purpose of this auxilliary heater is to compensate for side losses from the main heater, and in operation it is used to adjust the temperature of C, C to be the same as that of B,B. In this way the border C,C serves as a guard ring for the center square B,B. The effectiveness of such a device in causing lines of heat flow to be normal to the surface B,B and in making these lines pass through the test sample without any refraction is completely discussed by Van Dusen, and shall not be considered here. With the copper plates in place on each side of the heater, the whole

system is screwed together and forms one complete unit which will be referred to as the "heating or hot" plate. From the spacing of the winding and the resistance of the constantan ribbon per unit length, the resistance per unit area of heater can be computed. Knowing the current through the heater, the heat generated per unit area in unit time can be computed. For the particular heater used, the number of calories generated per second per sq. cm. for unit current is 0.00104. Since the temperature coefficient of resistance for constantan is very small, this value will be practically constant throughout the range of the experiments (from 20° C to about 130° C).

The samples, whose heat transmission properties are to be studied, are placed one on each side of the heater. "Water or cold" plates W,W are then placed next to the samples as shown in Figure **1**. These plates consist of square sheets of copper, $\frac{5}{32}$ inches thick and 24 inches on a side, on one face of which are soldered $\frac{1}{4}$ inch square copper tubes, laid parallel to one edge and set off at intervals of $\frac{3}{4}$ inch. These tubes extend across the whole plate, and the ends on each side fit into a manifold, which is also soldered to the same face of the copper plate. Van Dusen discusses the construction of such plates. Water, whose temperature is regulated by a thermostatic device and maintained at about room temperature, is allowed to flow through these tubes, maintaining the smooth copper surface, which is flush against the test sample, at a constant temperature. One of these cold plates is nickel-plated and highly polished on its smooth surface. This nickel-plated surface together with the nickel-plated center piece of the heater

are used in the experiments on air for reducing the radiation.

The whole system, including the hot plate, samples under test, and cold plates, is enclosed around the edge by a layer of cork, K,K four inches thick, and then clamped into a wooden frame G,G. In this way the system is made perfectly rigid and can be carried without disturbing the various parts. Two small wheels at H serve as an axis about which the apparatus can be rotated. These wheels rest on a narrow support provided with levelling screws at each end. The end J is supported from above. By adjusting the screws at H, the axis of rotation can be made horizontal, and with an adjusting screw at J the apparatus can be inclined at any desired angle.

For measuring the angle of inclination, an instrument combining a level with a protractor was constructed. The level and protractor both rotate as a unit about an axis through a steel plate. This plate is provided with a flat base which carries an indicator point for the protractor scale. When the base is placed on a horizontal surface the protractor is set so that the 90° angle coincides with the indicator point. Then the level is set in a horizontal position. This means that the 0° angle coincides with the horizontal line. With this adjustment, the instrument is ready for use. For, by placing the flat base on any surface, and rotating the protractor so that the level is horizontal, the angle read off at the indicator point gives the angle of inclination of this surface with the horizontal.

The surface of the tubes on the cold plate, as has been mentioned above, are flat and parallel to the opposite smooth side of the cold plate. Four holes, 1 mm. in diameter, are drilled through

the cold plate at a considerable distance from each other. With the experiments on air, the pressure inside is maintained at atmospheric pressure because of these holes, and the holes are also used to measure the distance between the hot and cold plates. This is accomplished by passing a drill rod, 1 mm. diameter, through the holes and measuring the distance between the hot plate and some convenient point on the cold plate. Then, knowing the distance between this point and the smooth surface of the cold plate, and applying a slight correction arising from the deviation of the line of the rod from the normal, the exact width of air space is known.

By means of the tightening screws in the wooden frame, the plates are set parallel, and then by placing the leveling instrument on the copper tubes, the plates are made horizontal or inclined at a desired angle. All these adjustments are made after the whole apparatus is in place. The leveling instrument is sensitive to 0.1° angle, and all adjustments are made to that degree of accuracy.

The temperature measurements are made entirely by means of copper-constantan thermocouples. The difference in temperature between hot plate P and the cold plate [illegible] is obtained by means of a differential couple whose constantan wire is soldered at one end to the center point of P, and the other end to the center of the corresponding cold plate. Copper leads are soldered to the edge of the inner hot plate, and to the cold plate, using the copper plates as part of the leads. Since the absolute temperature of the water plate is known, the absolute temperature of the hot plate is determined.

For measuring the difference in temperature between plate P, and the guard ring G, four short pieces of constantan wire are soldered across the narrow air space separating the plates, one in each corner. With copper leads soldered to each plate, a differential couple is obtained for observing the temperature difference.

The power input into the main heater is obtained by measuring the voltage drop across a low standard resistance of 0.006709 ohms in series with the heater, and by measuring the drop across 10 ohms of a total of 10,011 ohms in parallel with the heater. The current flowing through this high, shunt resistance, although very small, is taken into account in applying the corrections. The measurements of both the current and voltage were taken to check the resistance of the heater.

In measurements on the heat transmission through air, a layer of cork is placed on the copper side of the hot plate, and on the nickel-plated side is the air space surrounded by a layer of balsa wood F,F as shown in Figure I. In order to determine the heat transmitted through the air space, it is necessary first to determine the heat transmitted through the cork. This is obtained by taking two layers of cork, of equal thickness and alike in quality, and placing each on one side of the heater, and making a preliminary set of observations for the cork. For the quality and thickness of cork used in these experiments, the heat transmitted per sq. cm. per second per degree centigrade difference across the entire thickness is 0.000153 calories. This is for a mean temperature of the cork of 46.2° C.

For the variation of conductivity of cork with tempera-

ture the value given by Van Dusen is used in all computations. Expressed in terms of total heat transmitted per sq. cm. per second centigrade, difference across the entire thickness of cork, the value used for this variation with temperature is 3.4 x 10^{-6} per degree centigrade.

In the observations on air, one of the layers of cork E is placed as shown in Figure 1, and the air space is on the other side of the heating plate. The surfaces of P and [illegible] adjoining the air space are nickel plated, and highly polished, as has been mentioned above. It was deemed advisable to nickel plate, rather than silver plate the surfaces, because, although for a given polish silver has a much lower emissivity, silver tarnishes more readily than nickel, and the emissivity may becomes greater after sufficient exposure. The air space is the same in area as the center heating plate, 40 cm. square, and is surrounded by balsa wood. Within the air space are stretched copper-constantan thermocouples of number 32 wire. They are stretched parallel to the axis of rotation of the apparatus, with all copper wires extending out through one side of the space and the constantan wires through the opposite side. The junctions are arranged in the plane of symmetry of the apparatus. The wires are set off at given distances in a direction normal to the cold plate by means of the balsam wood F,F which consists of thin layers. In a direction parallel to the water plate the wires are separated two inches apart by means of the cork K,K which consists of small pieces one inch thick. In the

plane of symmetry, then, there are seven rows and seven columns of couples. The actual height of each row above the cold plate is as follows:

Row	Height in Cms.
1	0.98
2	2.01
3	2.79
4	3.56
5	4.35
6	5.13
7	6.18

Height of hot plate above cold plate = 7.05 cms. The distance of each column from the side a (see Fig. 1 of the air space is as follows:

Column	Distance in Cms.
1	4.76
2	9.84
3	14.92
4	20.00
5	25.08
6	30.16
7	35.24

The cold junctions of all these couples is the cold plate. Therefore each couple records the temperature at its position with regard to the temperature of the cold plate.

[illegible]. Observations with Horizontal Plates:-

The first series of observations on the air space were made with the hot and colt plates horizontal, and the hot plate above the cold plate. This condition should yield no convection currents, and the total heat transmitted should be by conduction. The temperature-gradients throughout the space, in a direction normal to the plates, should be constant.

Figures 2 ~~and~~ to 5, inclusive, show the distribution in temperature throughout the space, each for a different temperature-drop between the plates. Each individual line indicates the temperature along a column, the couples being arranged in rows and columns as described above. In Figures 2 and 3, the lowest line refers to the first column. In Figures 4 and 5 the line to the extreme right refers to the first column. The others are arranged in regular order.

For convenience of plotting, in figures 2 and 3 the abscissae are the same for all lines, but the zeros of the ordinates are shifted vertically. Above each set is drawn a line which is the mean for the particular set. The upper scales of ordinates and abscissae apply to the mean line. The ordinate scale for the individual lines is omitted to avoid confusion. In Figures 4 and 5 the ordinates are the same for all lines, while the zeros of the abscissae are shifted horizontally. The line to the extreme right in each figure, is the mean for the set. The abscissae for the set of lines is also omitted.

The lines are reasonably regular, considering the possible dis-

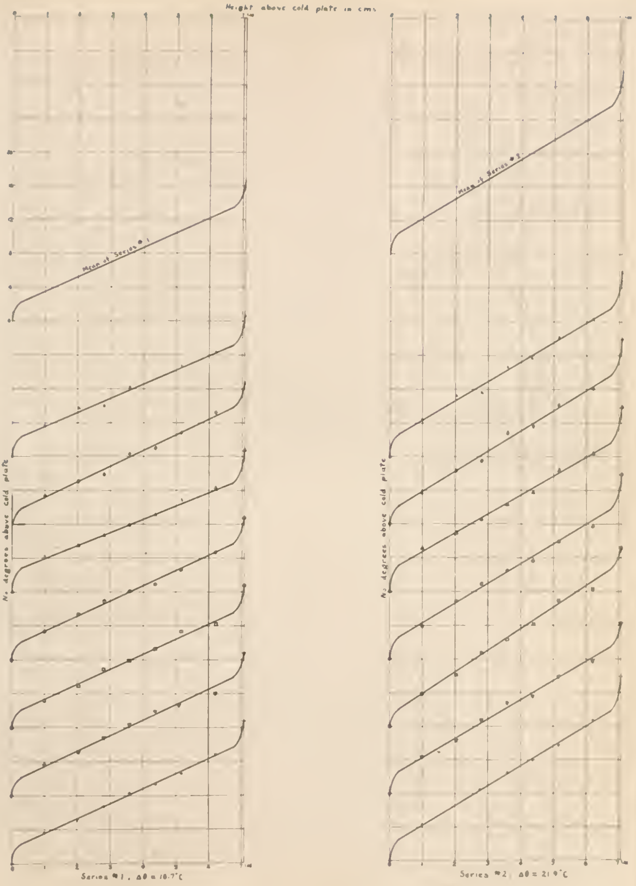
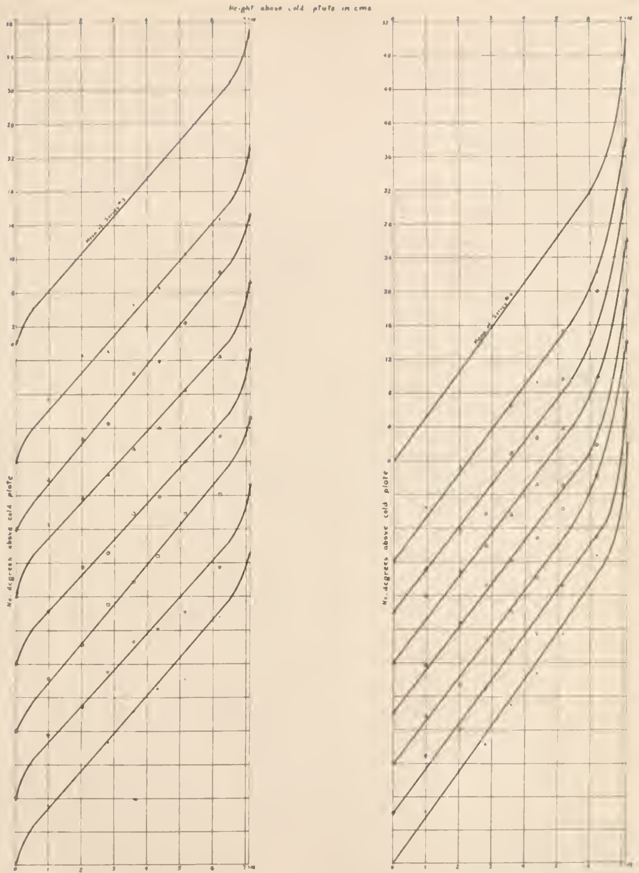


Fig. 2. Distribution in temperature along various columns. Plates horizontal.



Series #3 , Δθ = 37.3°C.

Series #4 : Δθ = 50.0°C

Fig. 3. Distribution in temperature along various columns. Plates horizontal.

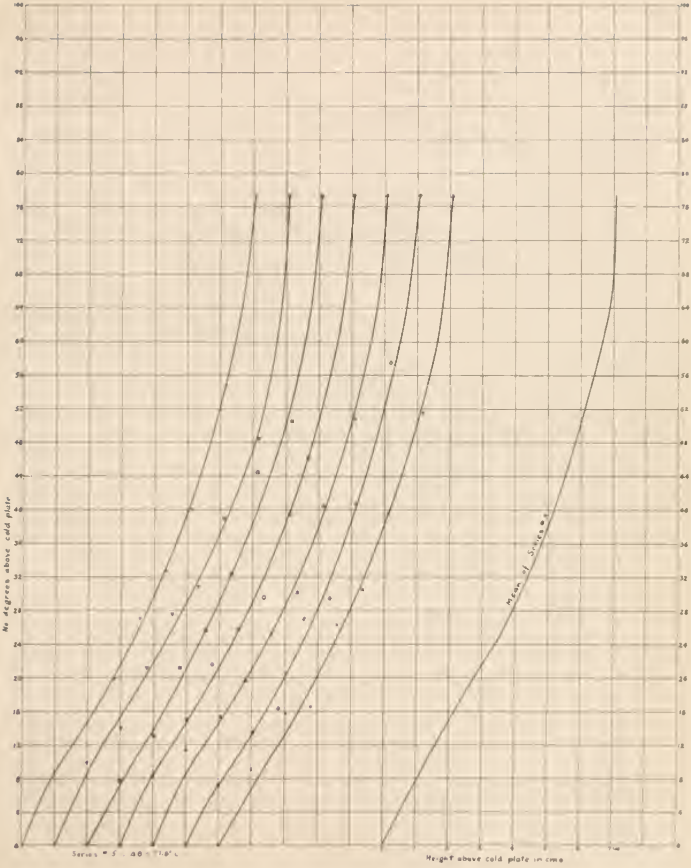


Fig. 4 Distribution in temperature along various columns . Plates horizontal

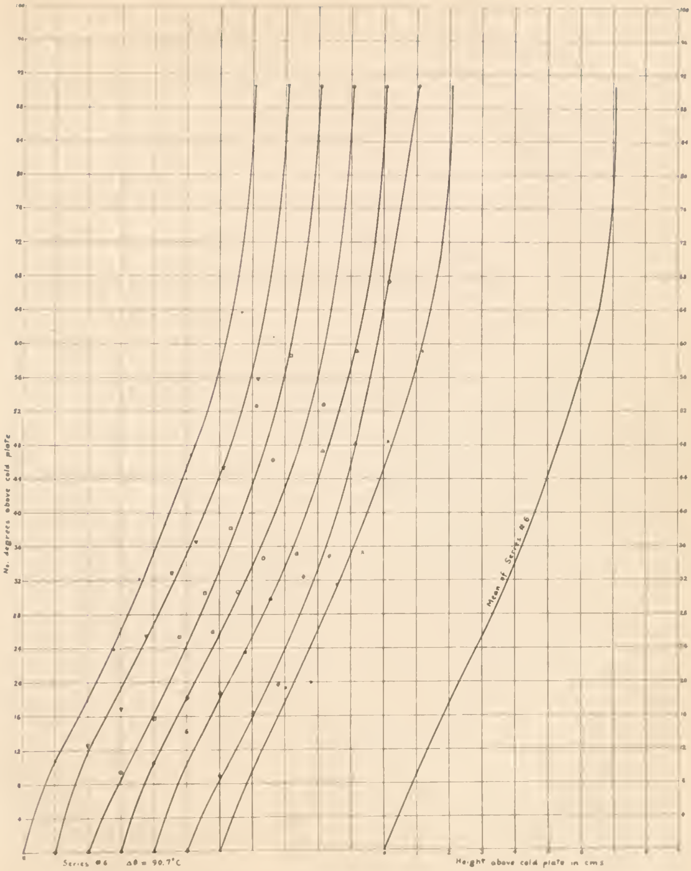


Fig. 5 Distribution in temperature along various columns Plates horizontal

alignment of the couples, which were difficult to get and keep in place very accurately. In Figure 3 the lines are straight throughout the space, except at the surfaces, where they become curved. In Figures 3,4 and 5, it is seen that as the temperature-difference between the plates increases the curvature and consequently the temperature-gradient increases at the surfaces, and gradually begin to reach far into the space. This would indicate that with an increase in the temperature between the plates, convection begins to manifest itself and become more distinct. Even with the smallest temperature-difference there must have been some convection close to the surface, in spite of the fact that care was taken to adjust the plates in a horizontal position to within 0.1° of angle.

In Table I are given the data for the different series of observations, from which is computed the thermal conductivity. θ_m is the mean temperature between the hot and cold plates, and $\Delta\theta$ the difference between the plates. h_0 is the total energy, in calories, transmitted across the air space per sq. cm. per second. h_r is the energy, in calories, transmitted by radiation per sq. cm. per second using the following relation for the radiation between two infinite, parallel plates:-

$$h_r = \frac{\epsilon}{2 - \epsilon} \; \sigma \left(\theta_2^4 - \theta_1^4 \right) \qquad (34)$$

ϵ is the emissivity, and for polished, nickel surfaces the values taken are $\epsilon = 0.0$[illegible]4 at 27°C, and $\epsilon = 0.0$[illegible]9 at 100°C. For intermediate temperatures the value of ϵ is obtained by interpolation. σ is the constant that enters the Stefan-Boltzmann law for the total radiation

Table I.

Air space with plates horizontal. Distance between plates, 7.08 cms.

Series	θ_m	$\Delta\theta$	h_o x 10^3	h_r x 10^3	(h_o-h_r) x 10^3	K x10^3	W
1	28.2	16.7	0.279	0.043	0.236	0.100	0.378
2	30.0	21.9	.437	.057	.380	.123	.608
3	36.6	37.2	.785	.107	.678	.129	1.084
4	42.1	50.0	1.290	.149	1.141	.161	1.823
5	59.4	77.6	2.220	.285	1.935	.177	3.100
6	65.6	90.7	2.870	.357	2.513	.196	4.010

from a black body, and the value taken for it is 1.35 x 10^{-12} calories per sq. cm. per second. $(h_o - h_r)$ gives the difference between the total energy transmitted and that by radiation, and if there were no convection this would be the energy transmitted per sq. cm. per second by conduction. K is the thermal conductivity computed from the relation

$$K = (h_o - h_r)\frac{7.08}{\Delta\theta} \qquad (24)$$

W is equal to the product of $(h_o - h_r)$ and the total cross-sectional area of the air space, 1600 sq. cms. In Figure 6, W is plotted against $\Delta\theta$, and in Figure 7 $(h_o - h_r)$ is plotted against the mean temperature θ_m. The dotted lines in both figures are the corresponding values taken from the paper of Van Dusen, which is a mean of several observers.

There is considerable difference between the values given by Van Dusen and those of the writer. It was the writer's primary in-

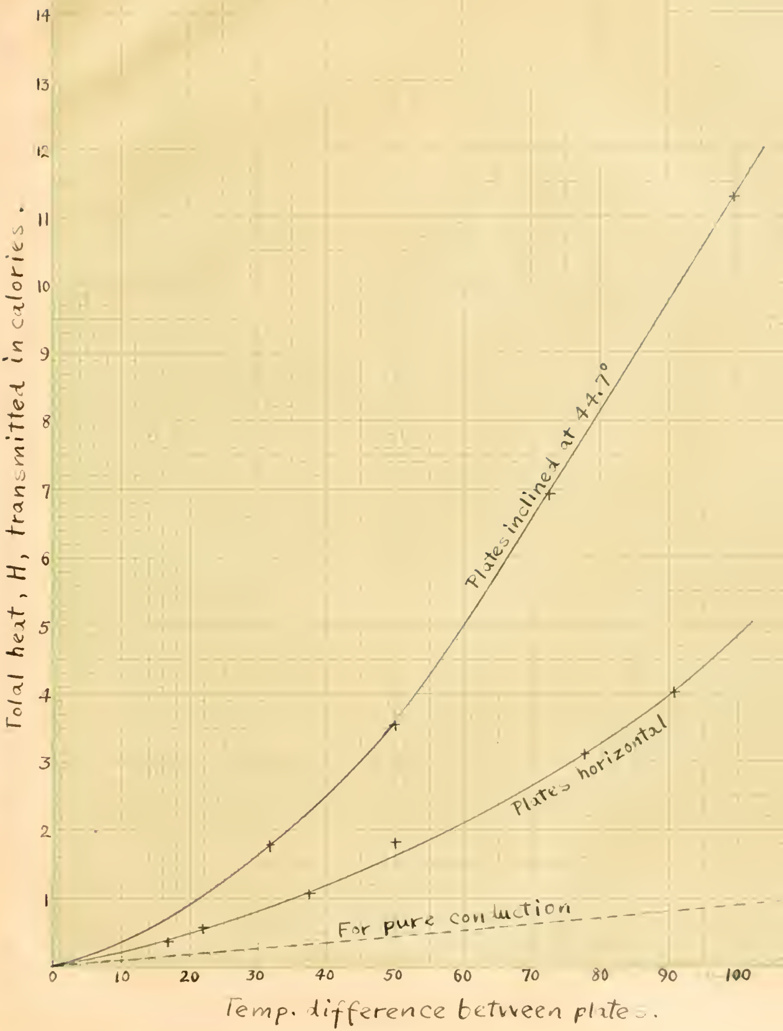


Fig. 6. Total heat transmitted for various temperature differenc

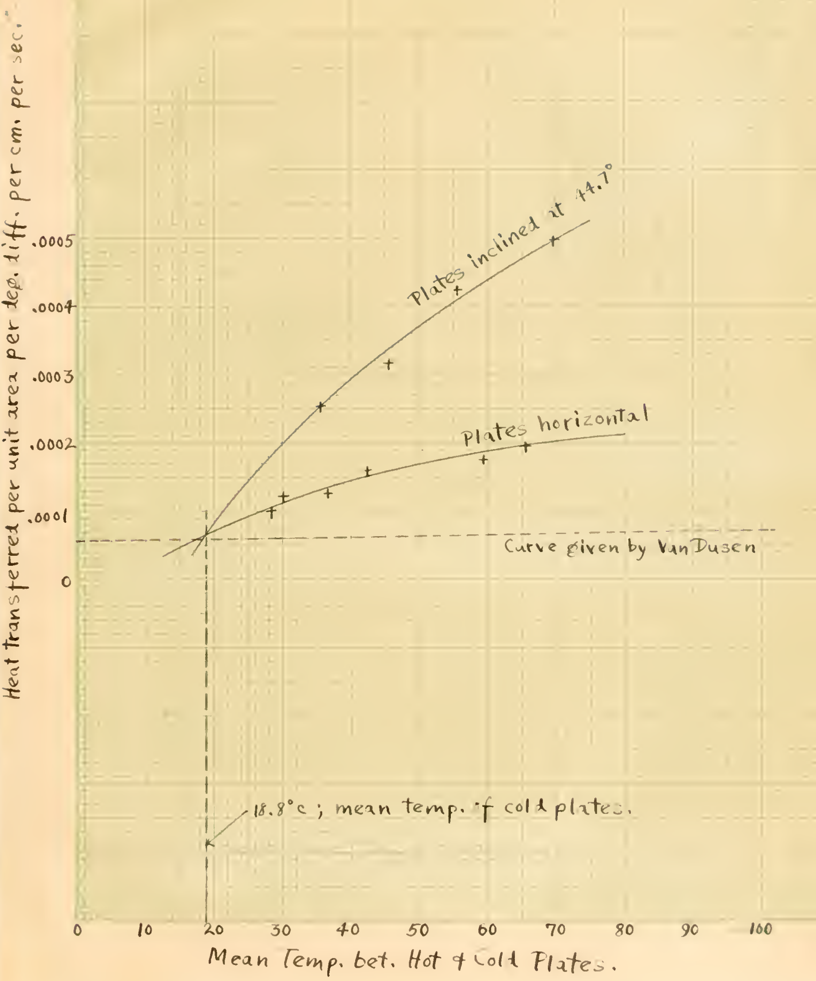


Fig. 7. Heat transferred per unit area per deg. diff. per cm. per sec.

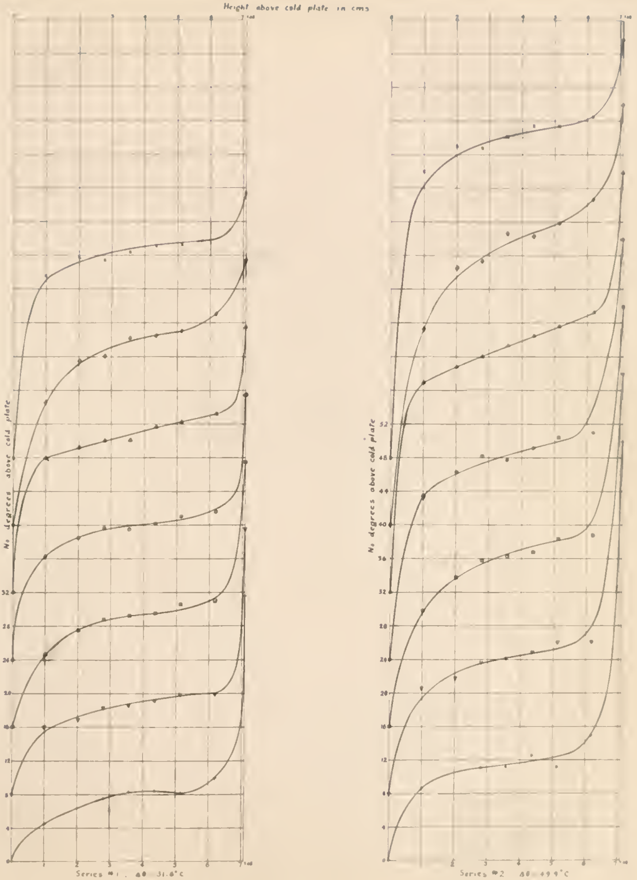


Fig 9. Distribution in temperature along various columns Plates inclined at 44 [illegible]°

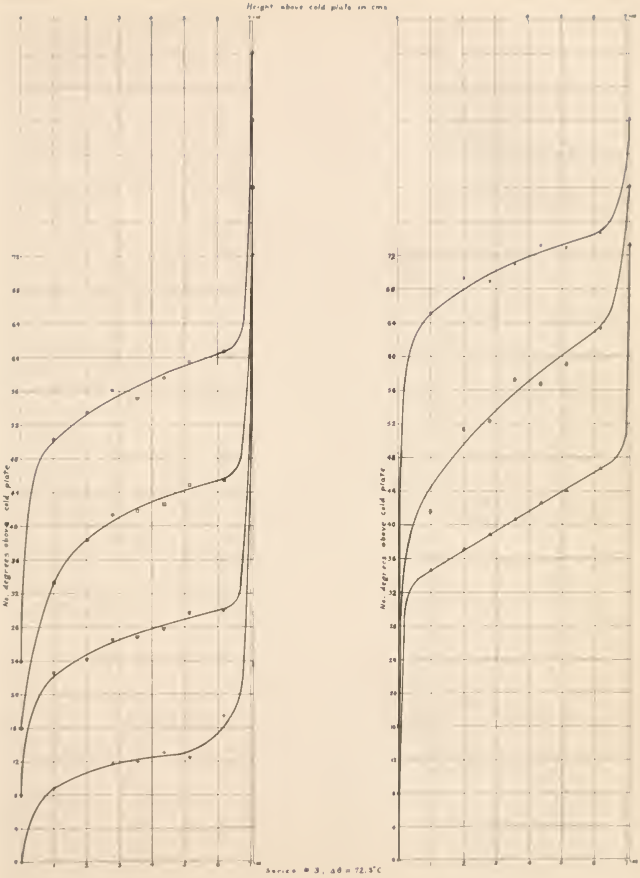


Fig 10. Distribution of temperature along various columns Plates inclined at 44.7°

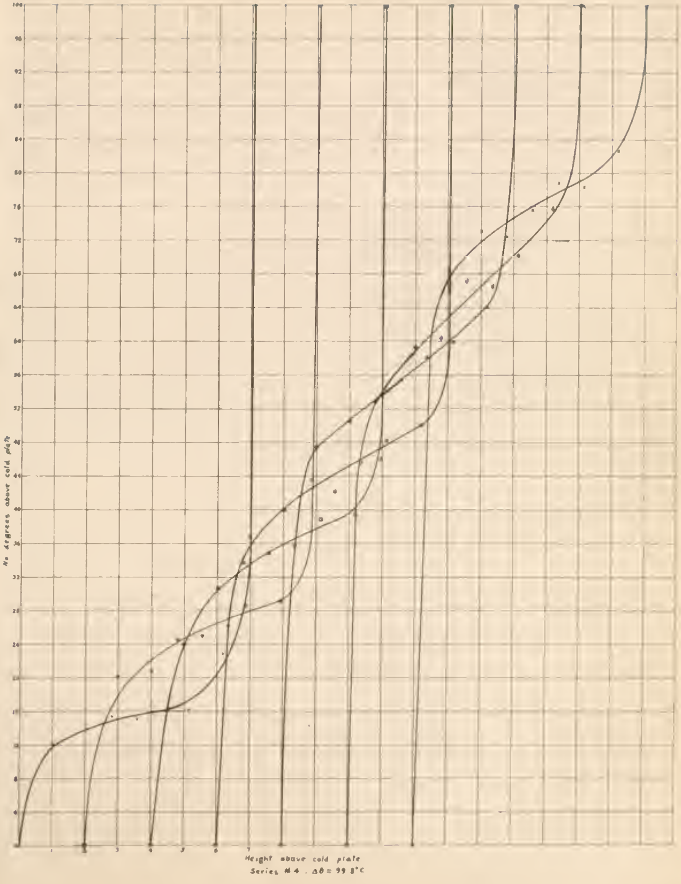


Fig 11. Distribution in temperature along various columns Plates inclined at 44.7°

tention to study the temperature distribution within the air space, and for this purpose it was necessary to make the space wider than has ordinarily been employed for measuring the conductivity. Hercus and Laby* employ a width of 0.655 cms. in their measurements on the conductivity of air, while the writer uses a width of 7.08 cms. The wider air space reduces the total conduction, so that stray effects such as convection become more pronounced. However, as the mean temperature θ_m approaches the temperature of the cold plate, making $\Delta\theta$ approach zero, the convection effects should become zero while the value for the conductivity should approach the true value. Figure 7 shows this to be the case. The mean temperature of the cold plate for the different series of observations is 18.8°C. The value for the conductivity of air at that temperature, as read off from the curve is 0.000065, which is a fair agreement with other observers.

3. Observations with the Plates Inclined at 44.7° with the Horizontal.

In these observations everything is set as above, except that the system as a whole is inclined at an angle of 44.7° with the horizontal, still keeping the hot plate above the cold plate. The distribution in temperature throughout the space is shown in Figures 9 to 11, inclusive. These lines are constructed in the same manner as those in Figures 2 to 5, inclusive. In Figures 9 and 10 the zero of the ordinates are shifted vertically, while in Figure 11 the zero of the

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* Proc. Roy. Soc. 95, pp. 190-210; Jan. 1919.

ordinates are shifted vertically, while in Figure 11 the zeros of the abscissae are shifted horizontally. The lines in Figure 10 all belong to one set, the lines to the left referring to columns 1, 2,3 and 4, and the lines to the right to columns 5,6 and 7.

It is interesting to note the change in shape of these lines compared to those taken with the plates horizontal. The temperature-gradients near the surfaces become greater, while within the interior region they decrease considerably. This is what would be expected physically, for close to the surface there is a layer of stagnant air through which heat is transmitted by conduction only. As the velocities increase, this layer becomes thinner, producing a steeper temperature-gradient. Within the interior, currents are well established and are transmitting the energy by convection, and but very little by conduction.

By means of Figures 9,10 and 11, isothermal lines are constructed as shown in Figures 12 to 15, inclusive. In Figures 12,13 and 14 the isothermal lines are drawn every two degrees. In Figure 15 they are drawn every 5 degrees. The abscissae are the distances in a direction parallel to the cold plate, while the ordinates are the heights above the cold plate. The origin corresponds to point d in Figure 1. The ordinates have been magnified in comparison with the abscissae so as to make the effects more visible. The various irregularities may not have existed in the actual case, but are merely a result of mag-

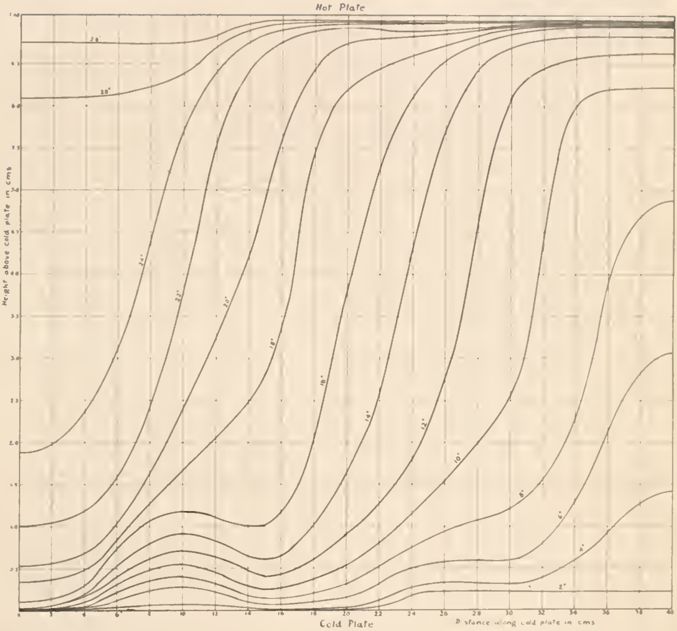

Fig 12. Isothermal lines for $\Delta\theta = 31.6°C$, plates inclined at 44.7° with horizontal.

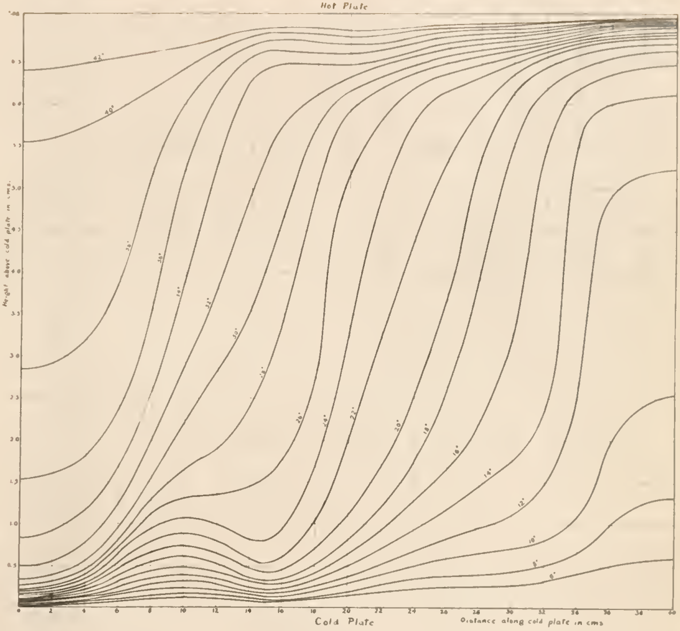

Fig 13. Isothermal lines for $\Delta\theta = 49.9°C$, plates incline at 44.7° with horizontal

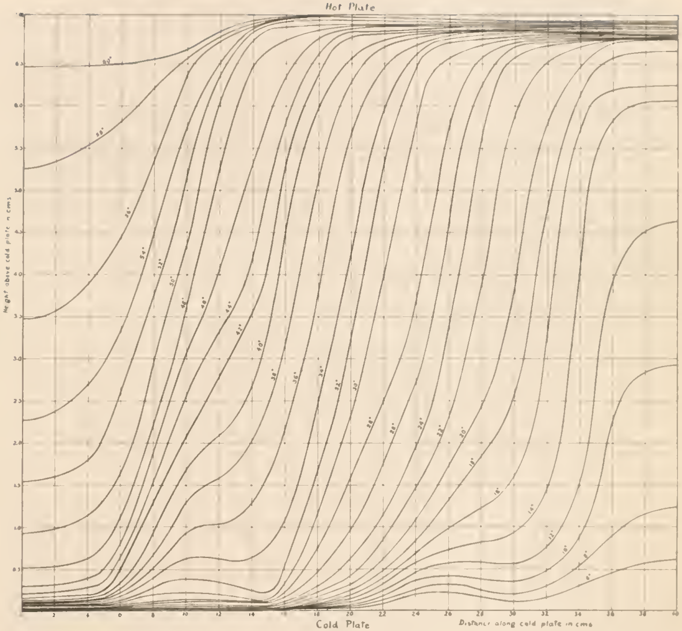

Fig 14 Isothermal lines for Δθ = 72.3°C plates inclined at 44.7° with horizontal

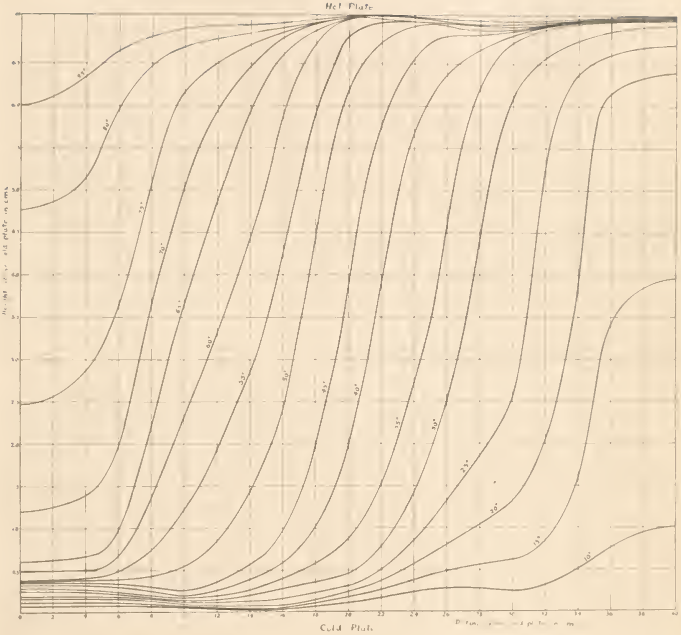

Fig 15 Isothermal lines for Δθ 99.8°C plates inclined at 44° with horizontal

nifying errors that were introduced into the previous curves. However, there is nothing to guide one in drawing ~~in~~ these lines, so it was thought advisable simply to follow the various points laid out in the space. In examining these curves one is to bear in mind that the plates are inclined, and that the point 40 cms. along the cold plate, and zero cms. above the cold plate is the lowest point in the system.

The curves show the general nature of the heat transfer. Conduction is greatest in the upper right-hand region and in the lower left-hand region. The same type of conduction is obtained for the different temperature-drops $\Delta\theta$ across the plates. In constructing these isothermal lines, the criteria of section 6 of the fundamental theory was followed.

With the aid of these isothermal charts the energy transferred by conduction per unit area per second across various points along a line parallel to the cold plate and lying in the plane of symmetry of the air space is determined. The results are shown in Figures 16 to 21, inclusive. They are obtained by computing the temperature-gradients at the various points in a direction normal to the cold plate, and multiplying these by the corresponding value for the conductivity. The abscissae are distances parallel to the cold plate, and the ordinates the energy transferred by conduction in calories per unit area per second. Each set of curves is for a given height

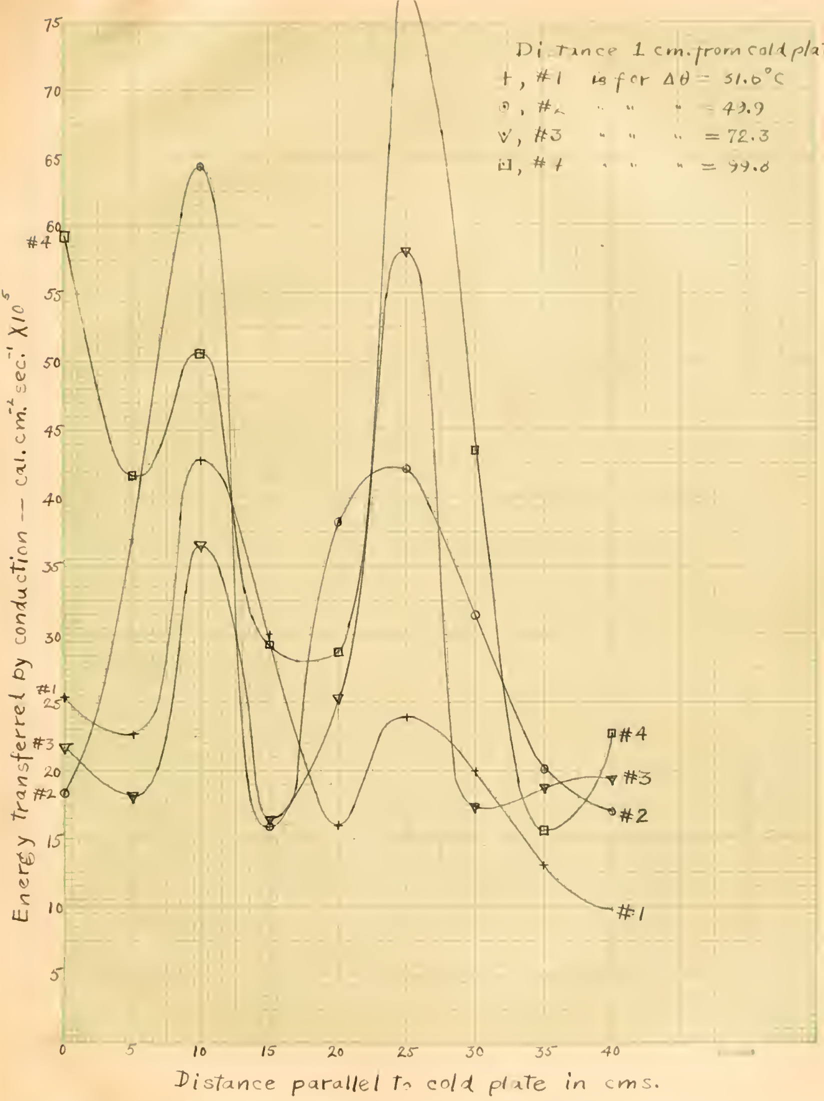

Fig. 16. Variation of conduction with position and temperature.

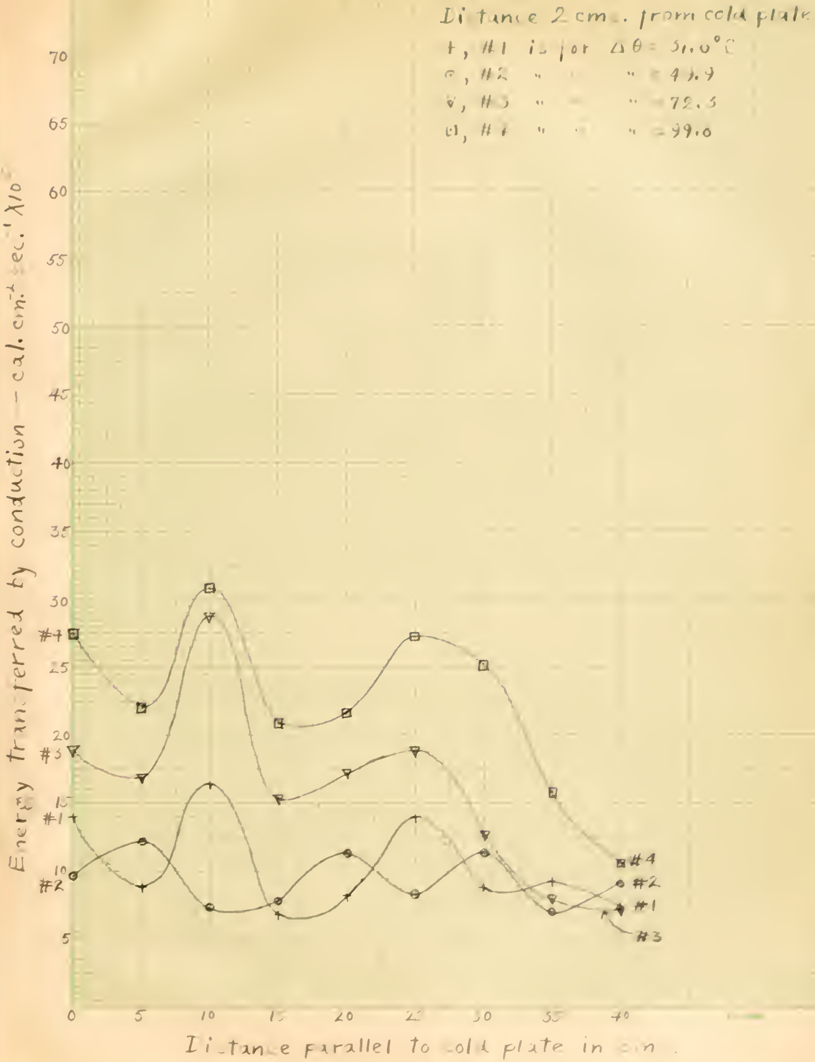

Fig. 17. Variation of conduction with position and temperature

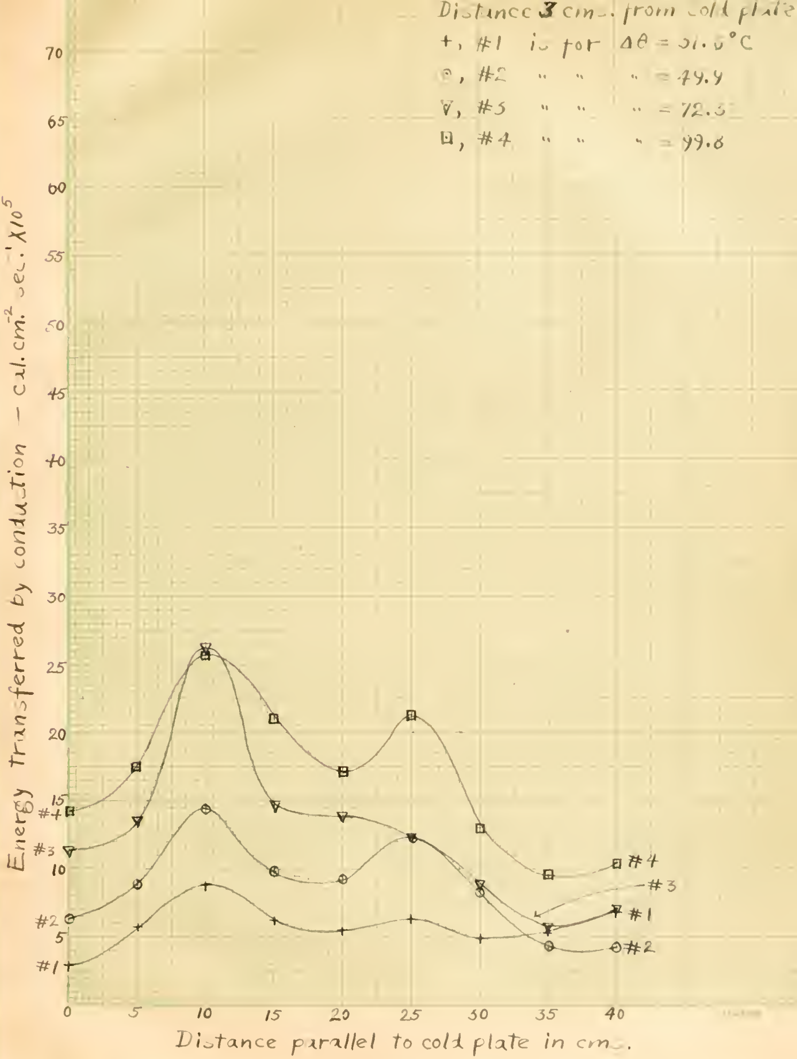

Fig. 18. Variation of conduction with position and temperature.

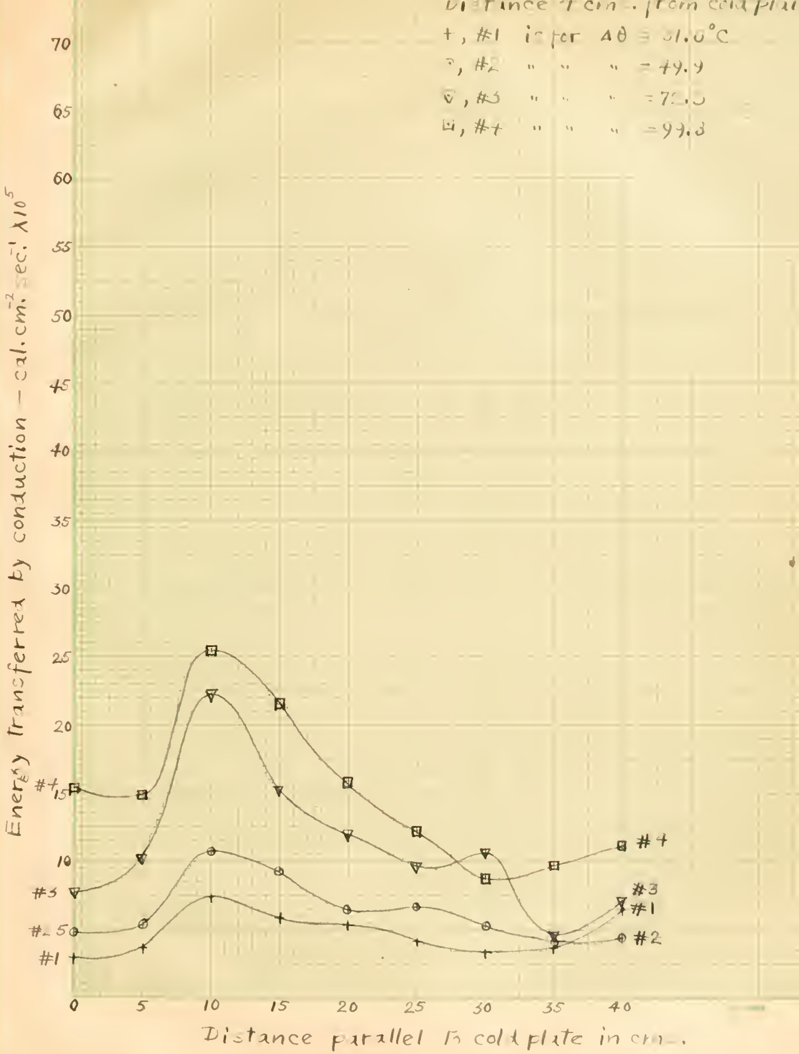

Fig. 19. Variation of conduction with position and temperature.

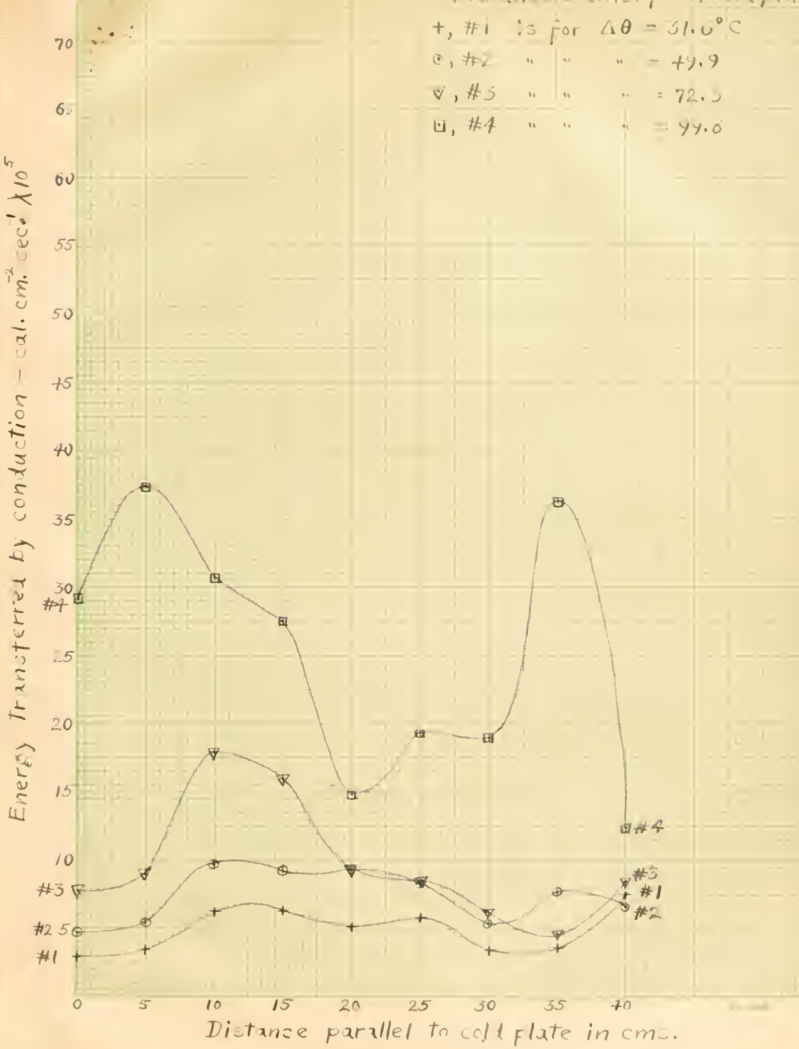

Fig. 20. Variation of conduction with position and temperature.

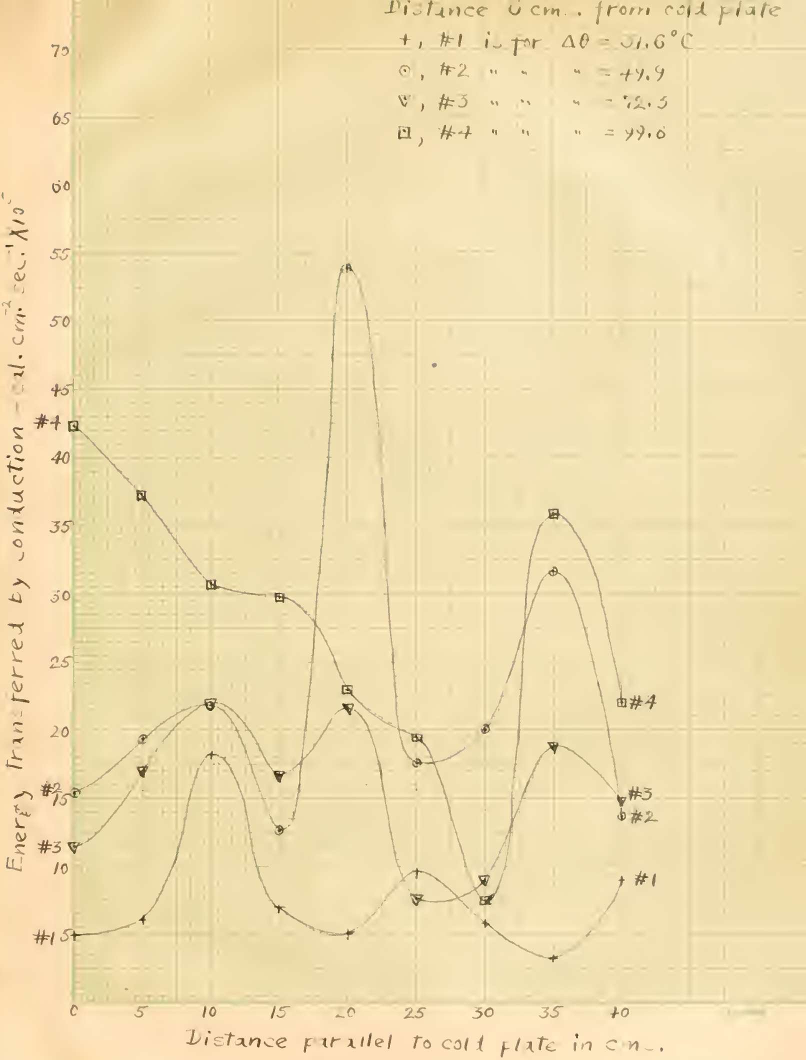

Fig. 21. Variation of conduction with position and temperature.

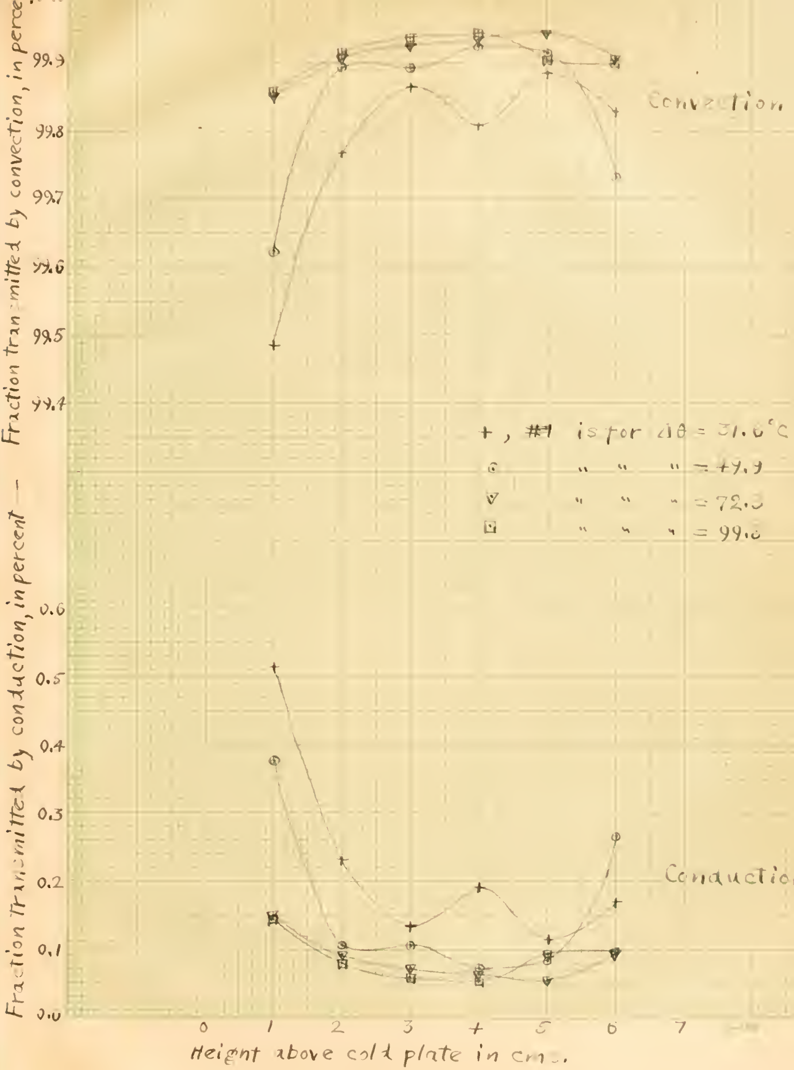

Fig. 22. Showing the relation between the heat transmitted by conducti
and convection across various planes and at different temperatur

above the cold plate, showing how the conduction varies with temperature. The area under each curve is the total energy conducted normally across the given line. These areas are measured with a planimeter, and they are plotted in Figure 22 as percentage of total energy transferred both by conduction and convection. In the lower half of Figure 22 is the percentage transferred by conduction, and in the upper half the percentage transferred by convection.

The results are rather surprising, for the amount transferred by conduction is less than one percent of the total transfer. Of course, this is for only one direction. In the actual case the phenomenon is very complicated. Conduction takes place normal to the isothermal lines, which are almost vertical with respect to the plates in the interior region. At the surfaces of the hot and cold plates the entire energy transfer is by conduction.

In Table II are given the data obtained for the series of observations with plates inclined at 44.7° from which is computed the total heat transmitted.

Table II.

Data taken with plates inclined at 44.7°. Distance between plates, 7.08 cms

Series	θ_m	$\Delta\theta$	$h_o \times 10^3$	$h_r \times 10^3$	$(h_o - h_r)\times 10^3$	$K\times 10^3$	u
1	35.6	31.6	1.21	0.088	1.12	0.214	1.795
2	45.5	49.9	2.38	0.156	2.22	.716	3.566
3	55.6	72.3	4.58	0.256	4.32	.424	6.972
4	69.5	99.8	7.46	0.410	7.05	.409	11.284

The notation is exactly the same as that used in Table [illegible].
K has no real significance in this case, but it was computed merely to compare its values with those for the horizontal plates. In Figure 6 is plotted K against $\Delta\theta$ for this case, and in figure 7 is plotted K against θ_m. The curves show a marked increase in transmission due to increased convection.

4. An Estimate of the Velocities:-

Equation (18) would yield a value for the absolute velocity of the stream at each point, provided its direction were known. In order to determine the magnitude of the velocities, four points were chosen for the case where the plates are inclined at 44.7° and with $\Delta\theta$ = 99.8°C. If x is the distance along the cold plate (see Figure 15) and y the height above the cold plate, the points at which the calculation is made are

	x	y
(1)	20 cms.	0.5 cms.
(2)	20	6.5
(3)	2	3.5
(4)	38	3.5

Points (1) and (2) are opposite the center, and approximately 0.5 cm. away from the cold and hot plates, respectively. Points (3) and (4) are near the center and 2 cms. away from the side walls. The direction of the velocity at each point is assumed parallel to the respective surface and with its sense so as to produce a clockwise circulation within the space.

If K be assumed constant, equation (16) can be written for the two-dimensional case in the form

$$|V| = \frac{K\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right)}{\Gamma\,(C_p - R)\,|\nabla\theta|\,\cos\left(V, \nabla\theta\right)} \qquad (25)$$

where $\Gamma = \frac{p}{R}$. For the case under consideration the pressure p is 1 atmosphere. Reducing all factors to c.g.s. units, the values taken from Kaye and Laby's tables are as follows:

$p = 1.013 \times 10^6$ dynes per sq. cm.

$C_p = 1.004 \times 10^7$ ergs

$R = 2.89 \times 10^6$ ergs for 1 gm. of air

$K = 2510$ ergs per sq. cm. per sec. per °C.

Therefore

$$\frac{K}{\Gamma\,(C_p - R)} = \frac{K}{p\left(\frac{C_p}{R} - 1\right)} = 0.00100$$

cos $(V, \nabla\theta)$ is determined graphically, as well as the first and second derivatives of θ, and $\nabla\theta$. The results of the computation are as shown below:

	Position		Velocity in cms. per sec.
	x	y	
(1)	20 cms.	0.5 cms.	1.01
(2)	20	6.5	3.81
(3)	2[illegible]	3.5	7.01
(4)	3[illegible]	[illegible].5	[illegible].10

It is seen that the velocities near the plates are greater than near the sides. Since the stream lines would be expected to lie closer together near the center of the plates as they would near the sides, this difference is quite reasonable. It is to be remembered that the stream lines, if drawn within the space would not be symmetrical with respect to the geometry of the space. For, the plates are in the actual experiment inclined at 44.7° with the horizontal, and therefore the mean density of the air would correspond to a point below the geometric center. This has the effect of crowding the stream lines below this center and spreading them out above the center. The above values should only be considered as an approximation, giving the order of magnitude of the velocities.

5. The Experimental Results considered as a Two-Dimensional Case:-

The question may arise whether the above results would actually apply to a two-dimensional system, and if sufficient data were obtained would lead to a solution of equations (20) and (23). Parallel to the plane of symmetry, within which all measurements are made, there are two rigid surfaces, one on each side and at 2 cms. from this symmetrical plane. If (x,y) are the coordinates within the plane of symmetry, and z the coordinates normal to this plane, there will be viscosity forces within the plane equal to

$$2\eta\frac{\partial u}{\partial z} \quad \text{and} \quad 2\eta\frac{\partial v}{\partial z}$$

per unit area, where (u,v) are the velocities in the plane. If these forces are negligible, the above results can be considered as applying to a system of two dimensions.

That this is the case, can be seen from experiments performed by Griffiths and Davis,* on natural convection produced along a vertical hot plate. They measure the velocities at different points along and away from the hot plate by means of a hot-wire anemometer, and find that for distances of about 2 cms. and more from the hot plate the velocities are no longer affected by the surface. In the above case the plane of symmetry is 20 cms. from each end. Therefore $\frac{\partial u}{\partial z}$ and $\frac{\partial v}{\partial z}$ can safely be placed equal to zero. Further evidence on this point is shown by the work of Stanton, Marshall and Bryant*. In studying the velocities of fluid passing through circular pipes, where the velocities are much larger than any experienced in the writer's work, they find the velocity close to the surface to be practically zero, and the velocity increases very rapidly as they pass away from the wall. The velocity-gradient becomes quite small at the center of the pipe. For the writer's case the velocity-gradients $\frac{\partial u}{\partial z}$ and $\frac{\partial v}{\partial z}$ would surely be negligible.

6. Conclusions:-

The problem treated above is of intrinsic value, and the results can have numerous applications. At the present time great strides have been made in improving the heat insulating properties

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* Report No. 9 of the Engineering Committee of the Food Investigation Board, Dept. of Scientific and Industrial Research. England, 1[illegible].

* Proc. Roy. Soc. A, 97, pp. 413-434, 1920

of building walls. It is a well recognized fact that air spaces within the walls improve the insulating properties, but the knowledge on which one can determine the most efficient type of construction is very meagre. It was with the view of completing this gap in our knowledge of insulation, that this work was primarily undertaken. But the problem is quite fundamental, and should be treated as one of pure Physics.

For building material the emissivity is quite large, possibly of the order of 0.80, so that one might expect considerable radiation. But of the other two factors, conduction and convection, the latter plays a surprisingly important rôle, as has been shown in this paper. In continuing this work it will be necessary to study the effect of the dimensions of the air space on each factor. This information should lead one to construct an air space which shall satisfy certain requirements, and shall have the maximum insulating properties. Of course, such information can determine a construction for transmitting heat most efficiently, a problem which may occur in practice very often.

There is a great advantage ultimately to obtain a solution, even if it be a rough approximation, of the fundamental equations which have been considered in this paper. This will place a mass of detailed information into a single formula, and will lend itself readily to direct computation. For practical applications this would be ideal.

In concluding, the writer wishes to express his gratitude to Dr. J.S.Ames and Dr. [illegible] Van [illegible] for their valuable criticisms and suggestions.

Biographical Note.

Joseph Louis Finkelstein, son of Reuben Boaz and Rose (Temchen) Finkelstein, was born in Russia on October 5, 1893. He attended the public and high schools of Boston, Massachusetts. He graduated from the Mass. Inst. of Technology, receiving the degree of Bachelor of Science in 1915. From July 1916 to January 1919 he was connected with the U.S. Bureau of Standards, holding the positions of Laboratory Assistant and Assistant Physicist. After leaving the Bureau he undertook graduate work at Harvard University, receiving his Master's degree in 1922. He was appointed Research Fellow in Mechanical Engineering at Harvard for the year 1921-22, and was engaged in research on the Joule-Thomson effect in superheated steam. In July, 1922 he was appointed Associate Physicist in the Bureau of Standards. He attended the Johns Hopkins University during the year 1922-23.